Speedable left-c.e. numbers

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Abstract. A left-c.e. real number α is ρ-speedable if there is a computable left approximation $a_0, a_1, \ldots$ of α and a nondecreasing computable function f such that we have $f(n) \geq n$ and

$$\liminf_{n\to\infty} \frac{\alpha - a_{f(n)}}{\alpha - a_n} \leq \rho,$$

and α is speedable if it is ρ-speedable for some $\rho < 1$. Barmpalias and Lewis-Pye [JCSS 89:349–360, 2016] have implicitly shown that Martin-Löf random left-c.e. real numbers are never speedable. We give a straightforward direct proof of this fact and state as open problem whether this implication can be reversed, i.e., whether all nonspeedable left c.e. real numbers are Martin-Löf random. In direction of solving the latter problem, we demonstrate that speedability is a degree property for Solovay degrees in the sense that either all or no real numbers in such a degree are speedable, and that left-c.e. real numbers of nonhigh Turing degree are always speedable. Furthermore, we obtain a dichotomy result: by definition, left-approximations of nonspeedable real numbers are never speedable, while for any speedable real number all of its left approximations are ρ-speedable for all $\rho > 0$.

Keywords: left-c.e. real numbers · Martin-Löf-randomness · Solovay reducibility.